Joint Framework for Signal Reconstruction using Matched Wavelet Estimated from Compressively Sensed Data

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The compressed sensing problem can be mathematically modeled as the following relation, $\mathbf{y} = \mathbf{A}\mathbf{x} + \zeta$. Here $\mathbf{y}_{M\times 1}$ and $\mathbf{x}_{N\times 1}(N > M)$ are the measured and original signal respectively, $\zeta_{N\times 1}$ is additive white Gaussian noise and $\mathbf{A}_{M\times N}$ is the measurement matrix. Subscript denotes the dimension of signal. If the original signal $\mathbf{x}$ is known to be sparse in the wavelet domain $\mathbf{W}$, it can be recovered in noise free case using optimization framework as below:

$$\tilde{\mathbf{x}} = \min_{\mathbf{x}} \; ||\mathbf{W}\mathbf{x}||_1 \text{ subject to } \mathbf{y} = \mathbf{A}\mathbf{x}. \tag{1}$$

Any existing wavelet can be used for the sparsifying domain but wavelet matched to a given signal provides best representation of a given signal and hence, provides better reconstruction in CS compared to any existing wavelet. No method exist in literature for estimation of matched wavelet that can be utilized at the same time for efficient reconstruction of signal from compressively sensed data. In this work, we address this problem.

Our method consists of three stages. In stage-1, we obtain coarse data estimate, $\tilde{\mathbf{x}}$ from partial samples by solving (1) using a standard wavelet. In stage-2, we analyze this signal using the same wavelet. We assume that the detailed coefficients, $\tilde{\mathbf{d}}_{-1}$, obtained using this known wavelet are noisy version of the actual detail coefficients that could be obtained with the matched wavelet, $\mathbf{d}_1$ i.e. $\tilde{\mathbf{d}}_{-1} = \mathbf{d}_{-1} + \eta$.

We subsample $\tilde{\mathbf{d}}_{-1}$ using i.i.d. Gaussian random matrix $\mathbf{\Phi}$ and rewrite above equation as below:

$$\tilde{\mathbf{d}}_{-1,sub} = \mathbf{\Phi}\tilde{\mathbf{d}}_{-1} = \mathbf{\Phi}\mathbf{d}_{-1} + \theta, \tag{2}$$

where $\theta = \mathbf{\Phi}\eta$. This will allow some degrees of freedom to extract sparser solution $\hat{\mathbf{d}}_{-1}$, which we recover using the following optimization framework:

$$\hat{\mathbf{d}}_{-1} = \min_{\mathbf{d}_{-1}} \; ||\tilde{\mathbf{d}}_{-1,sub} - \mathbf{\Phi}\mathbf{d}_{-1}||_2^2 \text{ subject to } ||\mathbf{d}_{-1}||_1 \leq \epsilon. \tag{3}$$

Now matched wavelet can be estimated using the same approach as in [1] by replacing $\mathbf{d}_{-1}$ with $\hat{\mathbf{d}}_{-1} - \mathbf{d}_{-1}$ in equation (9) of section III-A. In stage-3, we reconstruct original data from measured sub-sampled data with matched wavelet estimated in stage-2 using equation (1).

References

[1] N. Ansari and A. Gupta, "Signal-matched wavelet design via lifting using optimization techniques," in *Digital Signal Processing (DSP), 2015 IEEE International Conference on.* IEEE, 2015, pp. 863–867.

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