Physics Inspired CS based Underwater Acoustic Channel Estimation

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***Abstract*—We propose real-time channel tracking for underwater acoustic communications under dynamic sea conditions. The key idea is to employ sophisticated sparse sensing techniques that are cognizant of stable or slowly time-varying channel components against a transient background. Shallow water acoustic channel is generally challenging to track under moderate to rough sea conditions. This is primarily due to non-stationary highly transient elements within the channel delay spread resulting from rapidly fluctuating multipath arrivals from unpredictable surface wave reflections. The proposed channel estimation method exploits two channel characteristics: (i) Inherent sparsity of the time-varying channel in the two-dimensional dual (Fourier) domain; and (ii) Relative dominance of the direct arrival and slowly varying multipath arrivals against the otherwise non-stationary channel impulse response. Specifically, we utilize this apriori information to compressed sensing (CS) framework and thus, achieve channel sensing cognizant of time-frequency localization across significant channel taps. Numerical evidence based on data-driven channel ground truths are presented.**

***Index Terms*—Underwater acoustic channel estimation, compressed sensing**

I. Introduction

Underwater acoustic channel estimation in shallow water depths is a challenging problem due to rapidly fluctuating transients in the acoustic channel impulse response. Transmitted signal undergoes non-stationary reflections at the moving sea surface and rough sea bottom before being received via multiple paths at the receiver [1, 2]. These non-stationary reflections along with unpredictable surges of energy due to surface focusing events [3] render the channel delay spread challenging to localize in time, frequency, and sparsity. The goal of this work is to track the time-varying delay taps by exploiting physics-based channel characteristics such as the relative stationarity of direct and slowly varying multipath arrivals with respect to more transient multipath effects.

Figure 1 shows the two-dimensional (2-D) Fourier transform of shallow water acoustic channel over experimental field data collected at 15 meters depth and 200 meters range under moderate to rough sea conditions. Delay refers to the delay taps constituting the channel impulse response at a given time instant on the x-axis. In a medium range shallow water acoustic channel, there are two bands of interference besides the direct arrival: (i) the primary multipath interference dominated by single surface reflections, and (ii) secondary multipath interference dominated by multiple bounce reflections between moving sea surface and rough sea bottom. Additionally, sparsely distributed high-energy events such as surface wave focusing occur in both bands, thus rendering channel tracking in this paradigm exceptionally difficult.

Sparse sensing techniques (see e.g. [4-14]) for tracking the shallow water acoustic channel in medium ranges face two related challenges: (i) Non-stationary and rapidly time-varying delay taps, and (ii) Non-stationary temporal fluctuations of the support sparsity itself [5]. Furthermore, directly applying sparse sensing techniques suppresses detection of smaller channel delay taps, which are typically more persistent and hence, easier to predict, and often serve as a build-up to high-energy transients. Thus, there is a compelling need to bridge the gap between real-time detection of high-energy transients with high-precision tracking of stationary albeit smaller delay taps.

A. Background Motivation and Key Innovations

In this work, we propose a physics-driven approach to dynamic channel sensing by bridging concepts from acoustic communications and sparse optimization. The key idea is to employ sophisticated sparse sensing techniques that are cognizant of relatively stationary channel components against a non-stationary background. We draw upon recent work on designing suitable input signal dictionaries in the 2-D Fourier domain for MIMO transmission and signaling recovery [15]. We noted in [15] that the channel is sparse in the 2-D Fourier domain. Additionally, we exploit the relative importance of direct (line of sight) and slowly varying multipath arrivals (e.g. single surface bounce reflections without focusing phenomena) to robustly assess the channel. Specifically, we do not impose any sparse sensing of the most dominant and stable channel elements that manifest in the low Doppler frequency ranges in the 2-D Fourier domain. Numerical evidence demonstrates that the proposed approach achieves lower estimator error over conventional compressing sensing techniques [16, 17].

II. System Model and Related Art

In this section, we present the system model and review recent work on underwater acoustic channel estimation [15]

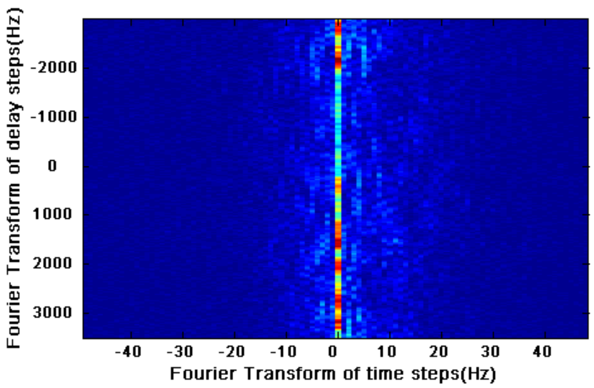

Fig. 1: 2-D Fourier Transform of Underwater Channel

that we extend significantly in this paper into a physics-driven compressive sensing framework.

We adopt the established model [1,2,4] of the time-varying shallow water acoustic channel using K delay taps and L Doppler frequencies, where the Doppler spectrum is used to localize time-variability of each channel delay tap. We also adopt the MIMO framework presented in [15] where the input signal is a complex exponential $x[i, f_k] = e^{j\frac{2\pi i f_k}{K}}$ sampling K possible delay frequencies $\{f_k\}_{k=0}^{K-1}$ across parallel sub-channels and L Doppler frequencies $\{f_l\}_{l=0}^{L-1}$ in the Doppler domain. To visualize the combined MIMO signaling and acoustic channel framework introduced in [15], first consider the noise-free scenario where the received signal $y[i, f_k]$ at time instant i in sub-channel f_k is mathematically modeled as:

$$
\begin{aligned}
y[i, f_k] &= \sum_{k=0}^{K-1} h[i,k]x[i-k, f_k] \\
&= \sum_{k=0}^{K-1} h[i,k]e^{j\frac{2\pi(i-k)f_k}{K}} \qquad (1)
\end{aligned}
$$

where h is the K-tap length time-varying channel specified at different time instants i. With algebraic manipulations, (1) can be re-written as

$$
y[i, f_k] = e^{j\frac{2\pi i f_k}{K}} \sum_{k=0}^{K-1} h[i,k]e^{-j\frac{2\pi k f_k}{K}} \qquad (2)
$$

and modified to obtain

$$
y_w[i, f_k] = y[i, f_k]e^{-j\frac{2\pi i f_k}{K}} = \sum_{k=0}^{K-1} h[i,k]e^{-j\frac{2\pi k f_k}{K}} \qquad (3)
$$

where $y_w[i, f_k]$ represents the received signal $y[i, f_k]$ weighted by $e^{-j\frac{2\pi i f_k}{K}}$. Equation (3) represents the one-dimensional (1-D) Fourier transform of the channel $h[i,k]$ along the second dimension, i.e., along the channel delay spread. Taking 1-D Fourier transform of (3) along the first dimension (i.e., the time variable i), we obtain

$$
\begin{aligned}
U[f_l, f_k] &= \sum_{l=0}^{L-1} y_w[i, f_k]e^{-j\frac{2\pi i f_l}{L}} \\
&= \sum_{l=0}^{L-1}\sum_{k=0}^{K-1} h[i,k]e^{-j\frac{2\pi i f_l}{L}}e^{-j\frac{2\pi k f_k}{K}} \qquad (4)
\end{aligned}
$$

Thus, $\mathbf{U}$ is the 2-D Fourier transform of $\mathbf{H}$, i.e.

$$
\mathbf{U} = \mathscr{F}\mathbf{H} \qquad (5)
$$

where $\mathbf{H}$ is the matrix form of time varying channel $h[i,k]$, $\mathbf{U}$ is the matrix form of $U[f_l, f_k]$, and $\mathscr{F}$ is the 2-D Fourier operator. The receiver estimates the channel $\mathbf{H}$ via 2-D inverse Fourier transform of $\mathbf{U}$.

Now consider the noisy scenario, where the signal received is modeled as:

$$
\begin{aligned}
\mathbf{U}_{obs} &= \mathbf{FH} + \mathbf{N} \\
&= \mathbf{U} + \mathbf{N} \qquad (6)
\end{aligned}
$$

where $\mathbf{N}$ refers to the complex white Gaussian noise matrix of size $L \times K$. Direct 2-D inverse Fourier transform of $\mathbf{U}$ in (6) will clearly not be able to estimate $\mathbf{H}$ precisely due to the noise process. However, (6) can be solved for $\mathbf{U}$ using the following optimization framework

$$
\min_{\mathbf{u}} ||\mathbf{u}_{obs} - \mathbf{u}|| < \sigma \qquad (7)
$$

where $\mathbf{u}$ and $\mathbf{u}_{obs}$ are the vectorized forms of $\mathbf{U}$ and $\mathbf{U}_{obs}$ respectively, $||\mathbf{v}||$ denotes the l^2 norm of the vector $\mathbf{v}$, and σ is the standard deviation or the measure of the noise level present in the signal.

III. Physics-driven Acoustic Channel Estimation using Compressive Sensing

Prior art [15] presented above utilizes all samples of the observed signal $\mathbf{U}_{obs}$ for channel estimation and therefore, is extremely sub-optimal in exploiting the underlying sparsity of the shallow water acoustic channel. It also does not attempt to harness well-known underwater acoustic phenomena and is therefore, agnostic of overlapping bands of stationary and non-stationary elements within the channel delay spread. Fundamentally, the physics-agnostic framework in [15] as well as the current state-of-the-art in shallow water channel estimation (see e.g. [4,5,18] and references within) completely ignore the coexistence of non-stationary high-energy transients due to surface wave focusing [3] and relatively steady delay taps due to direct arrival and persistent multipath. Considering this complex interplay between these diverse physical processes within the same channel is critical to disambiguation of sparse and non-sparse channel characteristics which in turn, is crucial to successful deployment of sparse sensing techniques. Therefore, we propose a physics-driven compressive sensing framework that employs apriori information on relatively steady dominant channel elements to track transient high-energy channel elements with higher precision.

In subsection III-A, we discuss channel estimation using basic CS in the 2-D Fourier domain. Section III-B explains the proposed channel estimation using physics inspired CS method. Both these approaches exploit the sparsity of channel in the 2-D frequency domain.

A. Basic CS based Channel Estimation with Partial Sampling at the Receiver

The channel $\mathbf{H}$ can be estimated from its partially sampled Fourier transform using as explained in the sequel. Figure-1 clearly demonstrates the inherent sparsity of the channel $\mathbf{U}$ in the 2-D Fourier domain. Let $n = LK$ be the dimension of $\mathbf{u}$, the vectorized form of $\mathbf{U}$. We sample m points (where $m < n$) of $\mathbf{U}$ using the sensing matrix $\mathbf{\Phi}$ of size $m \times n$ as:

$$\mathbf{u}_1 = \mathbf{\Phi u} \tag{8}$$

where $\mathbf{u}_1$ represents the vector containing the sampled points of $\mathbf{u}$. In practice, owing to noise, we observe the signal

$$\mathbf{u}_{1,obs} = \mathbf{\Phi u} + \boldsymbol{\eta} \tag{9}$$

where $\boldsymbol{\eta}$ is additive white Gaussian noise of size $m \times 1$. The above system of equations is under-determined that can not be solved directly, and therefore, we employ CS based optimization to solve this problem. On modelling the estimation of channel $\mathbf{u}$ as Basis Pursuit Denoising (BPDN) problem [18], we write it mathematically as below

$$\min_{\mathbf{u}} ||\mathbf{u}||_1 \;\; subject\ to: ||\mathbf{u}_{1,obs} - \mathbf{\Phi u}|| < \sigma \tag{10}$$

The above equation can be solved with the MATLAB solver SPGL1 [19]. After obtaining $\mathbf{u}$ from the above optimization formulation, (5) is used to estimate channel $\mathbf{H}$.

B. Physics Inspired CS based Channel Estimation with Partial Sampling at the Receiver end

In the previous subsection we used basic CS to estimate the channel, but did not use any apriori information about the channel. We now include physics-driven apriori constraints within the sparse sensing framework to derive a constrained optimal solution that accounts for the different channel elements that contribute to its complex nature. The Doppler frequency column at $f_l = 0$ represents the direct line of sight communication in the 2-D Fourier domain of the channel ($\mathbf{U}$), and by design, is the most stable and dominant component of the channel. Also, the single surface bounce in the primary multipath will have persistent low-frequency dominant components. We use the apriori information on the known channel support at zero and low Doppler frequencies in CS framework for channel estimation. Specifically, we do not impose sparse sensing of the zero and first Doppler frequency components that manifest as the three center columns of $\mathbf{U}$.

Generalizing the above discussion, let us assume that T denotes the support of elements of vector $\mathbf{u}$ to be used as apriori information, i.e., support of $\mathbf{u}$ that contains dominant steady components of the channel. Instead of sparse sensing on entire $\mathbf{u}$, we propose to carry out sparse sensing on the subspace of dimension $|T^c| = n - |T|$, where $|T|$ denotes the cardinality of the set T. The problem is formulated as below

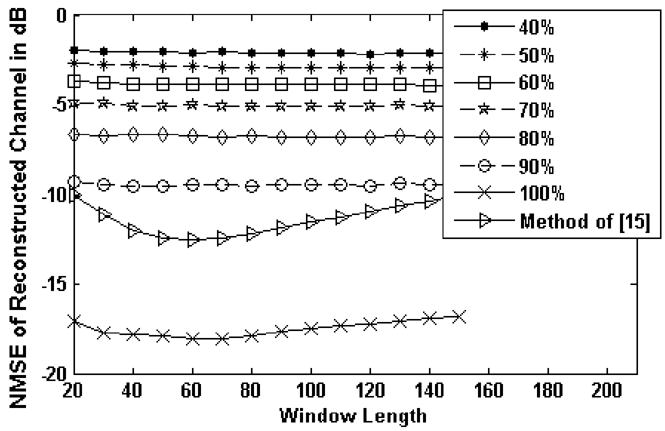

Fig. 2: Channel estimation using basic CS at 10dB

$$\min_{\mathbf{u}} ||\mathbf{u}||_1 \;\; subject\ to: ||\mathbf{u}_{1,obs} - \mathbf{\Phi}_R \mathbf{u}|| < \sigma \tag{11}$$

where $\mathbf{\Phi}_R$ is the restricted sampling operator that includes apriori information on T and hence, random sampling on T^c. On solving the above optimization problem, we obtain the estimate of $\mathbf{U}$ and hence, the channel $\mathbf{H}$.

IV. Experiments and Results

In this section, we demonstrate results using both basic CS and proposed physics-inspired CS as discussed in the previous section. We provide numerical evidence of superior performance against [15].

Experiments are performed on MATLAB 2013a platform on a 2.60 GHz i5 processor with 16 GB RAM for different window lengths with sampling ratios varying from 40% to 100%. For each window length and sampling ratio, results with different noise levels are averaged for 200 iterations. White Gaussian noise is added to the received signal. Channel signal-to-noise ratio (SNR) in dB is defined as below:

$$\text{Channel SNR} = 10\log_{10}\left(\frac{\frac{1}{LK}\sum_{i=0}^{L-1}\sum_{k=0}^{K-1}|h[i,k]|^2}{\sigma^2}\right) \tag{12}$$

where σ^2 represents noise variance. Normalized Mean Square Error (NMSE) in dB is used as performance metric to quantify the channel estimation results and is defined as below

$$NMSE = 10\log_{10}\left(\frac{\sum_{i=0}^{L-1}\sum_{k=0}^{K-1}|h[i,k] - \hat{h}[i,k]|}{\sum_{i=0}^{L-1}\sum_{k=0}^{K-1}|h[i,k]|^2}\right) \tag{13}$$

where $h[i,k]$ is the ground truth of the channel and $\hat{h}[i,k]$ represents the estimated channel.

Figures 2 to 5 present results with noisy channel SNR of 10dB and 5dB using method of [15], basic CS, and the proposed physics-inspired CS over different window lengths

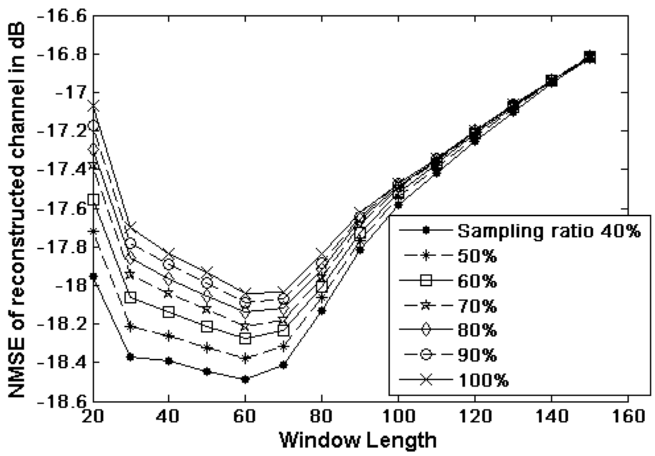

Fig. 3: Channel estimation using proposed method at 10dB

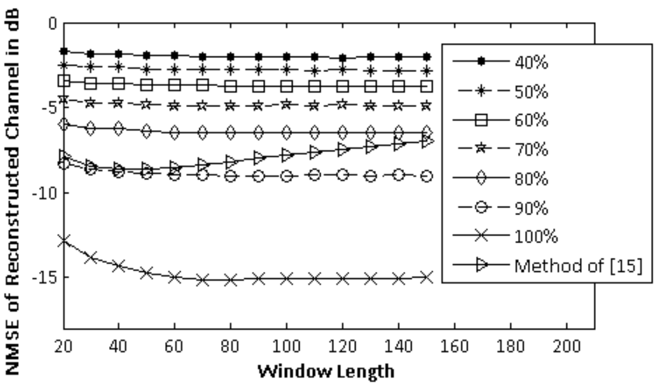

Fig. 4: Channel estimation using basic CS at 5dB

(in time) and sampling ratios of 40% to 100%. From these figures, we note that

1) 100% sampling ratio in basic CS performs better to method of [15] for both 10dB and 5dB. This indicates that with sparsity in the 2-D Fourier domain channel as considered in this work, better results are obtained over [15] where denoising of channel was performed by imposing l^1 norm constraint on the time domain channel.

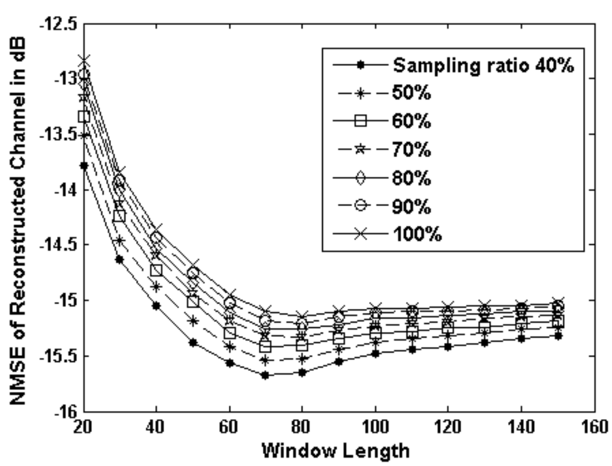

Fig. 5: Channel estimation using proposed method at 5dB

2) Better channel estimation results are obtained with the proposed physics-inspired CS framework compared to basic CS. At 100% sampling ratios both basic CS and physics-inspired CS are identical, physics-inspired knowledge helps with better denoising at lower sampling ratios.
3) NMSE reduces with the decreasing sampling ratio in the proposed method. Since the known apriori information samples in 2-D Fourier domain have high SNR compared to the rest of the sparse less amplitude coefficients, proposed framework with physics inspired CS recovers channel with greater accuracy.

V. Conclusion

We have proposed a method for underwater acoustic channel estimation using physics inspired CS framework. The physics-based method is designed based on knowledge of the dominant and relatively steady components of the channel. Specifically, we harness channel sparsity in the two-dimensional Fourier domain with prior information on the support of the dominant direct arrival and persistent multipath arrivals that are typically the result of non-focusing single surface reflections. From a physics perspective, this known support comprises of delay taps that contribute to both high-energy (direct arrival) and low-energy (persistent multipath) components that are crucial in the shallow water acoustic channel. Thus, we do not impose sparse sensing on this known support. The proposed framework enhances the underlying sparsity of the non-stationary channel delay spread that remains to be estimated. Comparison results show significant improvement in employing shallow water channel estimation algorithms with proposed CS over basic CS. We also observe better performance in physics inspired CS with decreasing sampling ratio as is expected from the reduced sparse subspace.

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