Underwater Acoustic Channel Estimation via CS with Prior Information

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***Abstract*—This paper proposes estimation of underwater acoustic channel in the delay-Doppler domain under dynamic sea conditions. The sparsity of the channel in the delay-Doppler domain is exploited via advanced compressive sensing (CS) technique to estimate the channel. The proposed use of CS with prior information takes care of relatively dominant but stable slowly time-varying channel component and the rapidly fluctuating high energy transients. Simulation results are presented on prior estimated channel of SPACE08 experiment, considered as ground truth, for the validation of the theory presented.**

Keywords: Underwater acoustic channel estimation, compressed sensing, Delay-Doppler domain

I. Introduction

Underwater acoustic channel has an unpredictable non-stationary and time-varying response due to rapidly fluctuating high energy transients caused by oceanic events such as surface wave focusing [1]. Also, the transmitted signal is received from multiple paths after it is reflected a number of times from the sea bottom and the moving sea surface [2], [3] leading to long time-varying delay spread, typically of the order of 100-200 delay taps. High-energy transients along with the long time-varying delay spread pose challenges in the tracking of underwater acoustic channel in real-time.

Nevertheless, several attempts have been made to estimate underwater acoustic channel. Methods for solving channel estimation problem include ray theory model [3], adaptive signal processing methods based on least squares [4], and channel estimation using multiple input multiple output (MIMO) framework [5]. Recently, sparse recovery methods are increasingly being used for tracking shallow water acoustic channel in medium ranges [6], [7], [8], [9], [10], [11] and are shown to provide better estimation than least square methods.

A major challenge in underwater acoustic channel estimation is high energy transient events that occur randomly. Since these transients are sparse in nature, these are best captured by sparse recovery methods. This establishes the utility of the sparse recovery methods. However, direct application of sparse sensing/recovery methods yield suboptimal results because they are not able to extract small amplitude channel delay taps that are typically steady and important often serving as a build-up to high-energy transients. Also, the sparsity of channel changes over time [6] that creates a challenge to the direct application of sparse recovery based methods. Hence, there is a need for a method that can detect steady component of channel as well as sparse and randomly occurring high energy transients. Motivated with the above, we designed a transmit dictionary in [10] and utilized it to estimate channel with sparse recovery method using sparsity of the channel in the 2-D time domain.

Compressed sensing falls under the umbrella of sparse recovery based methods where an under-determined system of linear equations is solved using sparsity of the signal in some domain as the prior knowledge. To the best of our knowledge, no existing method in literature had used CS *per se* for underwater acoustic channel estimation before the work of [11], [12]. In [11], [12], transmit dictionary designed in [10] is used for channel estimation in the compressive sensing framework exploiting the sparsity of the channel in the 2-D Fourier domain along with the knowledge of steady and transient channel components. These works were inspired by the CS based recovery in the Fourier domain in Magnetic Resonance Imaging (MRI) reconstruction.

However, we note that the underwater channel is sparser in the delay-Doppler domain as compared to the 2-D Fourier domain. Additionally, the channel is dense and has high energy in the lower Doppler frequencies, while it is sparse with comparatively less energy in the higher Doppler frequencies in the delay-Doppler domain. The current work is inspired by the above observations and the success of CS in underwater acoustic channel estimation in [11], [12]. This paper explores CS based method in the delay-Doppler domain for channel estimation. Further, variable density sampling has been proposed to exploit the sparsity and energy information as noted above.

The paper is organized in five sections. Motivation for using delay-Doppler domain for channel estimation using CS is presented in section-II. Proposed work is presented in section-III. Experimental results based on the proposed method are presented in section-IV and some conclusions are drawn in section-V.

II. Motivation for using delay-Doppler

Figure-1a shows the time domain representation of the shallow water acoustic channel estimated via non-convex mixed norm solver (NCMNS) algorithm [14] over experimental field data collected at 15 meters of depth from the sea surface and with 200 meters distance between the receiver and the transmitter under moderate to rough sea conditions in SPACE08 experiment [15]. Delay refers to the delay taps (in milliseconds) constituting the channel impulse response at any time instant represented by x-axis. Fig.1 shows the channel impulse response in the time domain, 2-D Fourier domain, and delay-Doppler domain.

From Fig.1, we note that the channel is sparser in the delay-Doppler domain compared to the 2-D Fourier domain. To validate the observation, a comparison of sorted magnitude of coefficients of the channel in the 2-D Fourier domain and delay-Doppler domain is presented in Fig. 2. The results have been shown over a window length of 10.8 msecs. For better visual clarity, a comparison of only 1000 largest coefficients has been shown in this plot. We observe that coefficients of the channel in the delay-Doppler decay more rapidly as compared to those in the 2-D Fourier domain. From [16], this observation indicates that the channel is sparser in the delay-Doppler.

Also, 98.5% of the total energy is occupied by 100 largest coefficients in the delay-Doppler, whereas only 82.0% energy is occupied by the same number of coefficients in the 2-D Fourier domain. This further affirms the claim. Hence, it is more appropriate to exploit the sparsity of the channel in the delay-Doppler domain as compared to the 2-D Fourier domain [17] and is the goal of this work. Particularly, we extend the formulation of [11], [12] to delay-Doppler domain and exploit the channel knowledge of steady component to implement variable sampling ratio CS with apriori information in the delay-Doppler domain for the channel estimation.

III. Proposed Work

We extend the system model proposed by us earlier in [11] to set-up the channel estimation problem in the delay-Doppler domain. In [10], we presented the idea of designing transmit dictionary via signaling of complex exponential signals $x[i, f_k] = e^{j\frac{2\pi i f_k}{K}}$, at time instant i over the k^{th} subband with a total of K parallel sub-channels corresponding to delay frequencies $\{f_k\}_{k=0}^{K-1}$. L denotes the maximum number of Doppler frequencies $\{f_l\}_{l=0}^{L-1}$ in the Doppler domain. We consider two cases for channel estimation: 1) when the signal is transmitted over the noise-free channel and 2) when the channel is noisy and hence, noise is added to the transmitted signal. The second case represents the practical communication scenario.

A. Noise-free scenario

In the noise-free scenario, signal $y[i, f_k]$ is received after linear convolution of the transmitted signal $x[i, f_k]$ with the time-varying channel impulse response $h[i, k]$ specified at time instants i with a maximum number of K taps. Mathematically, signal $y[i, f_k]$ is given as [12]:

$$\begin{aligned} y[i, f_k] &= \sum_{k=0}^{K-1} h[i, k] x[i-k, f_k] \\ &= \sum_{k=0}^{K-1} h[i, k] e^{j\frac{2\pi(i-k)f_k}{K}} \\ &= e^{j\frac{2\pi i f_k}{K}} \sum_{k=0}^{K-1} h[i, k] e^{-j\frac{2\pi k f_k}{K}}. \end{aligned} \quad (1)$$

On multiplying both sides of (1) with a multiplier $e^{-j\frac{2\pi i f_k}{K}}$ at the receiver, we obtain

$$y_w[i, f_k] = y[i, f_k] e^{-j\frac{2\pi i f_k}{K}} = \sum_{k=0}^{K-1} h[i, k] e^{-j\frac{2\pi k f_k}{K}}, \quad (2)$$

where y_w represents the received signal weighted by $e^{-j\frac{2\pi i f_k}{K}}$. Equation (2) represents one-dimensional (1-D) Fourier transform of the channel $h[i, k]$ along the channel delay spread. On computing the Fourier transform along the time variable i and inverse Fourier transform along the Delay frequency f_k, we obtain

$$\begin{aligned} \sum_{i=0}^{L-1} \sum_{k=0}^{K-1} y_w[i, f_k] e^{-j\frac{2\pi i f_l}{L}} e^{j\frac{2\pi k f_k}{K}} &= \sum_{i=0}^{L-1} h[i, k] e^{-j\frac{2\pi i f_l}{L}}, \\ D[f_l, k] &= \sum_{i=0}^{L-1} h[i, k] e^{-j\frac{2\pi i f_l}{L}}, \end{aligned} \quad (3)$$

where right hand side of (2) represents delay-Doppler of the channel. We consider, $D[f_l, k] = \sum_{i=0}^{L-1} \sum_{k=0}^{K-1} y_w[i, f_k] e^{-j\frac{2\pi i f_l}{L}} e^{j\frac{2\pi k f_k}{K}}$ in (2) above.

The above equation can be represented in matrix form as:

$$\mathbf{D} = \mathcal{F}\mathbf{H}, \quad (4)$$

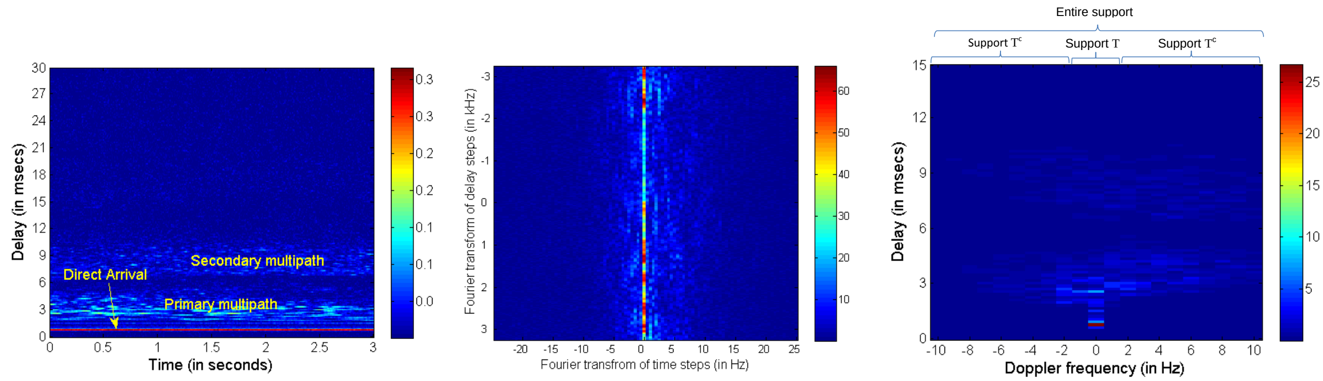
where $\mathbf{H}$ is the matrix form of time varying channel with dimension $L \times K$, $\mathbf{D}$ is the matrix form of delay-Doppler $D[f_l, k]$ with same dimension, and $\mathcal{F}$ is the Fourier transform operator that computes one-dimension Fourier transform along the time-variable. Channel can be estimated by computing inverse Fourier transform of the post processed received signal D in the noise-free scenario using the above relation.

B. Noisy scenario

In the noisy scenario, received signal after processing is:

$$\begin{aligned} \mathbf{D}_{obs} &= \mathcal{F}\mathbf{H} + \mathbf{N}, \\ &= \mathbf{D} + \mathbf{N}, \end{aligned} \quad (5)$$

where $\mathbf{N}$ denotes complex additive white Gaussian noise with dimension $L \times K$. Owing to the presence of noise in (5) above, it is not possible to estimate channel $\mathbf{H}$ via direct inverse Fourier transform of $\mathbf{D}$ similar to the noise free scenario. Hence, one has to involve some other signal processing methods to reconstruct the channel in the noisy scenario.



(a) Channel in 2-D time domain

(b) Channel in 2-D Fourier domain

(c) Channel in Delay-Doppler domain

Fig. 1: Shallow water acoustic channel of duration 3sec estimated via [14] from field data of SPACE08 experiment [15], plotted as a 2D image showing significant time-variability in primary and secondary multipath regions (in linear colorbar).

In accordance with the CS theory, we sense fewer components of the post-processed received signal, $\mathbf{D}$ at the receiver end. This can be achieved using:

$$\mathbf{D}_{sub} = \mathbf{\Phi}\mathbf{D} + \mathbf{N}, \quad (6)$$

where subscript '*sub*' denotes the subsampled signal and $\mathbf{\Phi}$ denotes the sub-sampling operator that randomly picks $S\%$ samples of $\mathbf{D}$, where $S = \lceil \frac{M}{LK} \rceil$, M is the number of sub-samples picked from $\mathbf{D}$, and $\lceil . \rceil$ represents the ceil function.

From Fig. 1b and 1c, it is noted that the lower Doppler frequencies that represent the slowly-varying components of the channel are the most dominant and contain most of the energy of the channel, while comparatively less energy is contained in the high frequency components. Also, it is evident from these figures that lower Doppler frequencies have dense high amplitude coefficients, whereas higher Doppler frequency components are sparser compared to lower Doppler. We use this information of sparse and non-sparse channel components for variable sampling in this work.

Let us denote lower Doppler frequencies by support T and higher Doppler frequencies by support T^c as shown in Fig.1c. As stated above, lower Doppler frequencies represent steady component of the channel and have high energy as compared to higher frequencies. Using this prior information about the channel, we retain all the samples in support T and do partial sampling in support T^c. This modifies (6) to include support T as follows:

$$\mathbf{D}_{sub,prior} = \mathbf{\Phi}_{prior}\mathbf{D} + \mathbf{N}, \quad (7)$$

where $\mathbf{\Phi}_{prior}$ represents the sub-sampling operator that does full sampling on support T and partial sampling on support T^c. $D_{sub,prior}$ is the result of sub-sampling that has used prior information about the channel. The above can be written in the vectorized form as below:

$$\mathbf{d}_{sub,prior} = \mathbf{\Phi}_{v,R}\mathbf{d} + \mathbf{n}, \quad (8)$$

where $\mathbf{d}_{sub,prior}$, $\mathbf{d}$ and $\mathbf{n}$ represent the vectorized form of $\mathbf{D}_{sub,prior}$, $\mathbf{D}$ and $\mathbf{N}$, respectively. $\mathbf{\Phi}_{v,R}$ represents the operator that does partial sampling in the vectorized form of delay-Doppler.

The problem of estimating channel can be posed as an optimization problem with the prior that the channel is sparse in the delay-Doppler domain. We formulate our problem as the LASSO (Least Absolute Shrinkage and Selection Operator) optimization problem [20]:

$$\underset{\mathbf{d}}{\operatorname{argmin}} ||\mathbf{d}_{sub,prior} - \mathbf{\Phi}_{v,R}\mathbf{d}||_2^2 \ \ subject\ to: ||\mathbf{d}||_1 < \tau, \quad (9)$$

where τ is the measure of sparsity of the channel in delay-Doppler domain. We solve the above via the MATLAB solver 'spgl1' [21], [22]. Equation (4) is used to recover channel in the time domain after solving (9).

IV. Experimental Results

Numerical results are presented in this section with the channel discussed in section-II as the ground truth. Experiments are performed for window lengths ranging from 3 msecs to 24 msecs with sampling ratios 40%, 70%, and 100%. We consider zero Doppler frequency as support T and use $\tau = 0.3\sqrt{LK}$ in these experiments. Results are generated for 200 Monte-Carlo simulations with white Gaussian noise being added to the received signal at 10dB and 5dB channel SNR. Channel estimation performance is quantified via Normalized Mean Square Error (NMSE) (in dB) given by

$$\text{NMSE} = 10\log_{10}\left(\frac{\sum_{i=0}^{L-1}\sum_{k=0}^{K-1}|H(i,k) - \hat{H}(i,k)|}{\sum_{i=0}^{L-1}\sum_{k=0}^{K-1}|H(i,k)|^2}\right), \quad (10)$$

where $\mathbf{H}$ and $\hat{\mathbf{H}}$ represent channel ground truth and reconstructed channel respectively. Figure-3 and 4 present results on channel estimation using the sparsity constraint on 2-D time-domain channel [10], using physics-inspired CS with sparsity constraint on 2-D Fourier transform of channel [11], and the proposed work. Fig.-3 and 4 present channel estimation results with noisy channel SNR of 10 dB and 5 dB, respectively. Comparatively improved performance of channel estimation is observed with the proposed work. This observation also shows that more is the sparsity, better is the recovery of the signal from its compressive measurements.

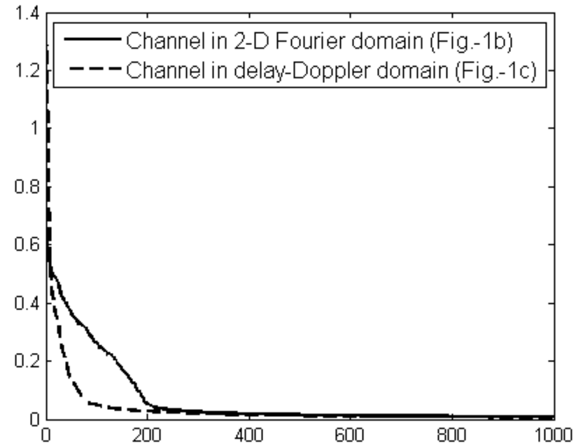


Fig. 2: Sparsity comparison of the channel in the 2-D Fourier domain and the delay-Doppler domain.

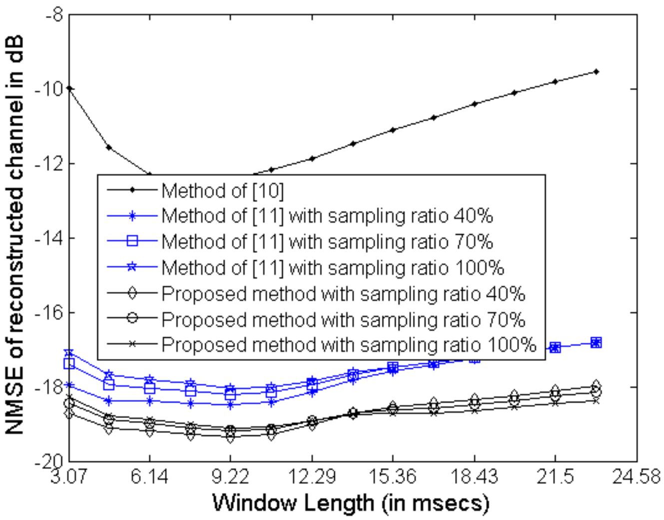


Fig. 3: Channel estimation using the proposed work at 10dB SNR

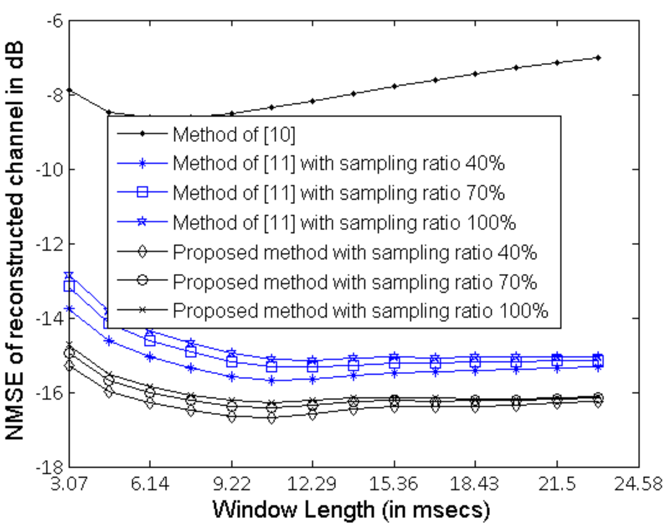


Fig. 4: Channel estimation using the proposed work at 5dB SNR

This is to note that better results are obtained at lower sampling ratios because at lower sampling ratios, ratio of higher amplitude positions (with higher SNR) to lower SNR samples is higher compared to 100% sampling ratio. This results into better channel recovery with the proposed CS with prior information set-up in the delay-Doppler domain.

V. Conclusion

Compressed sensing is utilized for underwater acoustic channel estimation in this work. Channel is recovered in the delay-Doppler domain using sparsity in that domain. Prior information about energy and sparsity along different Doppler frequencies is used for sub-sampling of delay-Doppler coefficients leading to variable density sampling in different Doppler frequency range. The proposed method is observed to perform better compared to channel estimation using CS in the time-domain or 2-D Fourier domain.

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